EMPIRICAL BACKGROUND PAPER OF THE STACO MODEL

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Wageningen University, Environmental Economics and Natural Resource Group

by Rob Dellink, Juan Carlos Altamirano-Cabrera, Michael Finus, Ekko van Ierland, Arjan Ruijs and Hans-Peter Weikard

1. INTRODUCTION

In this paper, the specification and calibration of the functions used in the STACO model are discussed. This paper is only meant as background information; for general information on the model and a discussion of the results, see Finus *et al.* (2003). The aim of STACO is to analyse coalition formation in global pollution control. The model can be used to determine which coalitions between (groups of) countries are stable and what will be the resulting effect on the stock of CO_2. The STACO model calculates the pay-off from CO_2 emission reductions for each possible coalition structure and compares the changes in pay-offs for individual regions from leaving or entering a coalition. In this way, the stability of the different coalitions can be assessed. It is assumed that members of a coalition maximize the aggregate pay-off of all coalition members together, while singletons maximize their own pay-off. Note that we only look at CO_2; other greenhouse gasses are, for simplicity, not included in the analysis.

Essential in the model is the choice of the time horizon. Given that damages due to climate change will only occur in the longer run (after 50 or 100 years), the time horizon should be sufficiently long. For that reason, the model covers the time period from the year 2011 to 2110. The STACO model captures the net present value of the stream of pay-offs generated between 2011 and 2110. Data for individual years between 2011 and 2110 are used to calculate the stock of CO_2 in 2110 and to calculate the net present value of benefits and abatement costs. All prices ($) are in 1985 US dollars[1].

In the following sections, first, the benchmark development of the CO_2 stock and the impact of abatement on this stock are discussed. Second, the different elements of the pay-off functions, *i.e.* benefits from abatement and abatement costs, are defined, and it is discussed how the input data for empirical calibration of the STACO model are constructed. Aggregation of the costs and benefits of abatement over time are discussed in the last section. A full list of symbols used and the final model equations are presented in Appendix 1.

2. NOTATION

The starting year for evaluation of the pay-offs is 2010. The model horizon is set at 2110. Let t denote all years in the model horizon, *i.e.* t=2011, 2012,…, 2110.

[1] In line with Ellerman and Decaux (1998).

The twelve regions considered are USA, Japan (JPN), EEC, other OECD countries (OOE), Eastern European countries (EET), former Soviet Union (FSU), energy exporting countries (EEX), China (CHN), India (IND), dynamic Asian economies (DAE), Brazil (BRA) and the rest of the world (ROW).[2] The regions are denoted in the parameters and variables with a subscript i; the set of regions is denoted by I.

3. EMISSIONS

For the calculations of emissions and the stock of CO_2 we have used the market scenario from the DICE model[3]. In the market scenario, there is no emission reduction, but the damages due to climate change are considered to have a negative impact on the global economy. Damages are expressed in monetary terms, as a result of a lower level of of gross world product (GWP, *i.e.* global GDP). This implies that, in this scenario, there is a feedback loop from the environment to the economy. Abatement efforts are endogenous in the STACO model. Therefore, the reference levels of emissions and stock of CO_2 have to be taken from a scenario with zero abatement, and are labelled as the uncontrolled level of emissions and uncontrolled stock of CO_2, respectively.

As argued below, we use a different damage function than DICE, and our estimate of the damage impacts on the economy in a situation without abatement should reflect the damages envisaged in the STACO model[4]. This involves rescaling the damage function in the DICE model to be consistent with the damage function used in the STACO model. To be precise, the parameter that governs the level of damages at a doubling of CO_2 concentrations is adjusted (see the discussion on the damage coefficients below).

The *adjusted market scenario* that is calculated in this way provides the best expectation of the development of the (uncontrolled) emissions and stock of CO_2 for the situation in which the regions do not react on climate change, and hence reflects the reference scenario for our model.

Global uncontrolled emissions (E_t) are assumed to be growing linearly over time:

$$E_{t+1} = E_t + d_E \qquad (1)$$

where d_E stands for the uncontrolled annual growth of emissions in Gigaton.

Note that the emission-intensity of the world economy (emissions per $ GDP) is not constant, due to several factors, including technological progress in the form of increases in energy-efficiency over time. Hence, an exponential growth of the economy (constant growth rate) can lead to a linear growth in uncontrolled emissions.

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[2] EEC includes the 15 nations of the European Union as of 1995. Other OECD countries (OOE) include among others Canada, Australia and New Zealand. Eastern European countries (EET) include among others Hungary, Poland, and Czech Republic. Energy Exporting Countries (EEX) include among others the Middle East Countries, Mexico, Venezuela and Indonesia. Dynamic Asian economies (DAE) include South Korea, Philippines, Thailand and Singapore. Rest of the World (ROW) includes among others South Africa, Morocco and much of Latin America and Asia. (For complete details see Babiker *et al.*, 2001)

[3] In this paper the original version of the DICE model is used (Nordhaus, 1994). Though there are more recent versions of the DICE model (*e.g.* Nordhaus, 2002), the original version has the advantage that this model specification, with its strengths and weaknesses, is widely known.

[4] Given the feedback loops between environment and economy, damages have an indirect effect on the level of emissions.

We calibrate the annual growth of emissions, d_E, such that the development of the uncontrolled *stock* of CO_2 fits as good as possible with the projection of the uncontrolled stock of CO_2 in our adjusted market scenario of DICE (using OLS regression):

$$E_{t+1} = E_t + 0.153 \quad (2)$$

The starting value for emissions in 2010 is taken from our adjusted market scenario of the DICE model (see above) and equals E_{2010} = 11.96 Gton. This path of emissions over time drives the development of the stock of CO_2. Figure 1 shows the DICE emission projection (adjusted market scenario) and the associated linear approximation for the situation without emission abatement. Though the fit for emissions is not very good, this linear approximation does provide a reasonable fit for the development of the stock of CO_2 as shown below.

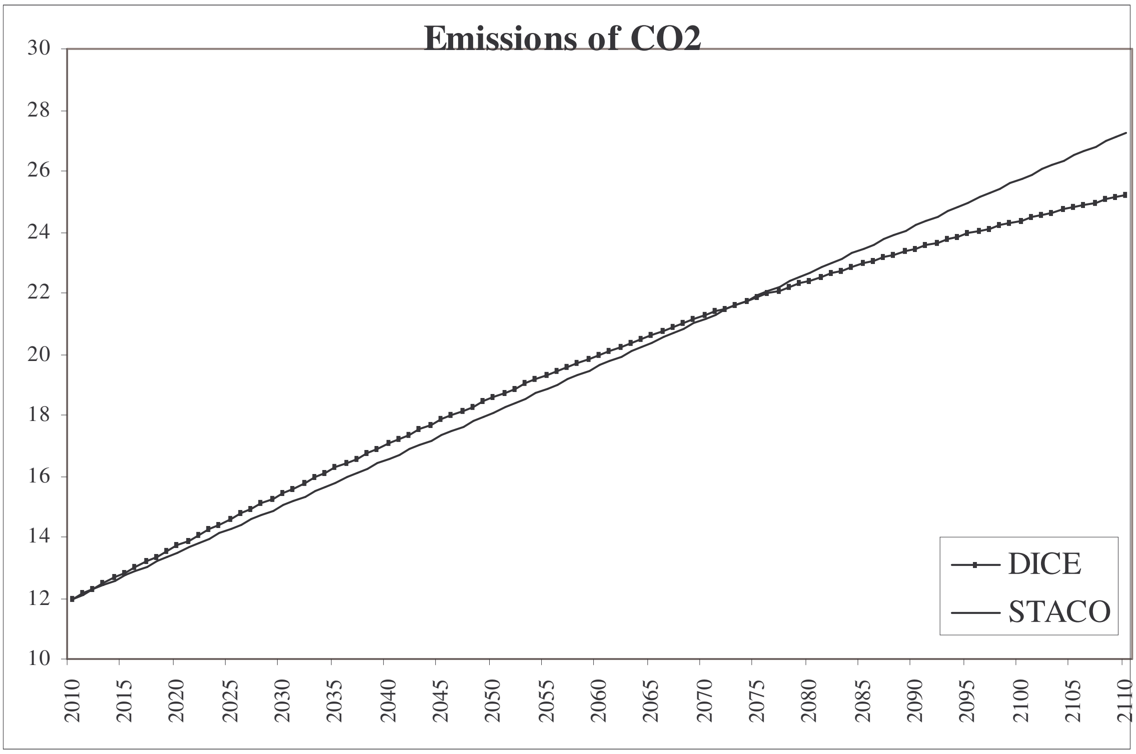

Figure 1. Uncontrolled emissions of CO_2 in Gigaton according to the adjusted market scenario of DICE and the STACO calibration.

Regional emissions are not given in DICE. The shares of the various regions are taken from Ellerman and Decaux (1998), the same source that we use for the abatement costs. Note that these regional emissions only serve as reference point to the model and do not influence the results. The calculated regional emission shares are given in the appendix.

4. Stock of CO_2

In this section, we derive the stock of CO_2 as a function of the uncontrolled emissions of CO_2 and the level of abatement. The analysis builds on Nordhaus (1994).

Since greenhouse gases are uniformly mixing in the atmosphere, the stock of CO_2 is only relevant from a global perspective. Let $M_t\left(q_t\right)$ give the stock of CO_2 in period *t* that results from a series of abatement levels $q_{2011},...,q_t$, using $q_t = \sum_{i \in I} q_{it}$.

The CO_2 stock in the year *t* is equal to the sum of the following components:

(i) the pre-industrial stock of CO_2 ($M_{pre\text{-}ind}$),

(ii) the part of the stock of CO_2 in 2010 which remains in the atmosphere in year *t*, and

(iii) the uncontrolled emissions minus the abatement for all regions between the years 2011 and *t* as far as they remain in the atmosphere in year *t*.

First, the pre-industrial stock of CO_2 is assumed to be an equilibrium level, *i.e.* this stock remains constant over time. Second, annually a part of the excess stock of CO_2 above the pre-industrial level decays. The decay factor, or the annual removal rate of carbon, equals 0.00866 (value taken from DICE). Third, emissions in the year 2010 are used as starting point for calculating the uncontrolled emission levels between the years 2011 and 2110. Some fraction of annual emissions remains in the atmosphere (set at 64%, based on the DICE model), and this fraction decays over time (with a factor 0.00866). The carbon sinks take up the remainder of the annual emissions. CO_2 stock levels refer to the stocks at the end of the year. For that reason, emissions and abatement in 2010 are already included in the stock of 2010, and emissions in the year *t* itself are included in the stock of year *t*.

This can be expressed as:

$$\begin{aligned} M_t\left(q_{2011},...,q_t\right) = {} & M_{pre-ind} + (1-0.00866)^{(t-2010)} \cdot \left(M_{2010} - M_{pre-ind}\right) \\ & + \sum_{s=2011}^{t} \left((1-0.00866)^{t-s} \cdot 0.64 \cdot \sum_{i \in I} \left(E_{i,s} - q_{i,s}\right) \right) \end{aligned} \tag{3}$$

As a reference point, the uncontrolled stock of CO_2, *i.e.* when there is no abatement, can be determined:

$$\begin{aligned} M_t\left(\mathbf{0}\right) = {} & M_{pre-ind} + (1-0.00866)^{(t-2010)} \cdot \left(M_{2010} - M_{pre-ind}\right) \\ & + \sum_{s=2011}^{t} \left((1-0.00866)^{t-s} \cdot 0.64 \cdot \sum_{i \in I} E_{i,s} \right) \end{aligned} \tag{4}$$

The uncontrolled stock can directly be calculated using (2). Using the DICE model we obtained the starting stock M_{2010} = 835 Gton, this leads to an uncontrolled stock of CO2 in 2110 of $M_{2110}(0)$ = 1585 Gton. The actual DICE calculations, using our adjusted market scenario, give a corresponding value of 1576 Gton.

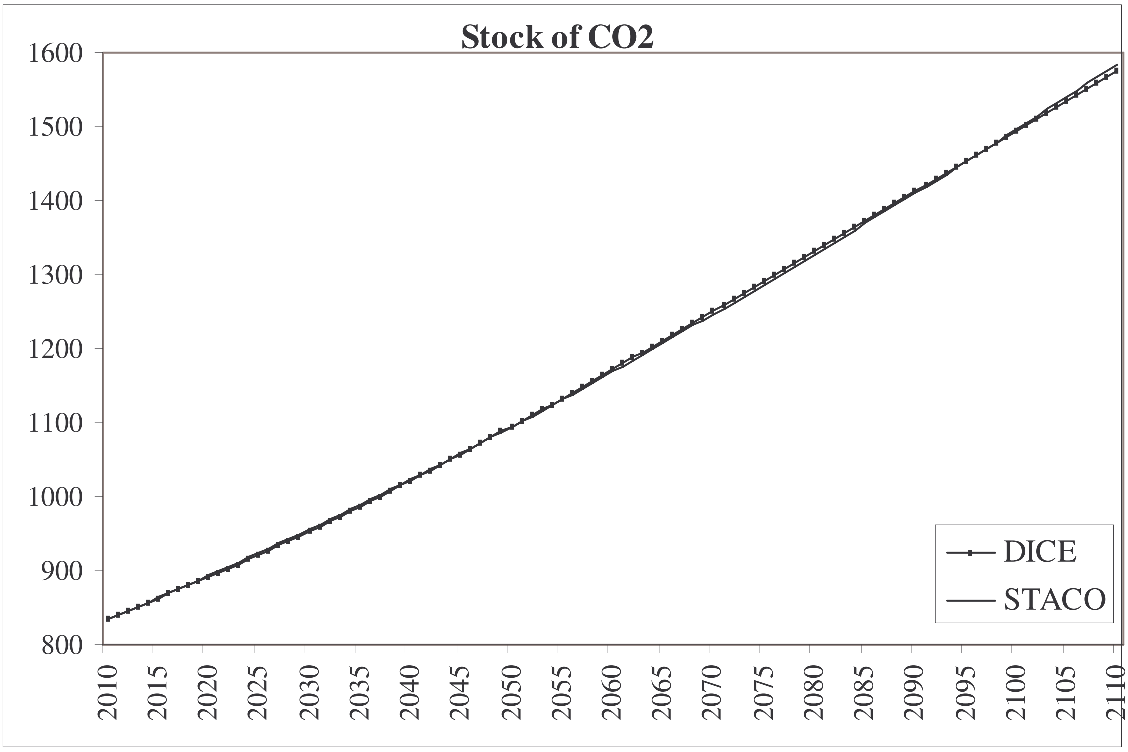

Figure 2. The uncontrolled stock of CO_2 in Gigaton according to the DICE market scenario and the STACO calibration.

Finally, the controlled stock for abatement levels $q_{2011},\ldots,q_t$ can be expressed as a function of the uncontrolled stock:

$$M_t\left(q_{2011},\ldots,q_t\right) = M_t\left(\mathbf{0}\right) - \sum_{s=2011}^{t}\left(1-0.00866\right)^{t-s}\cdot 0.64\cdot q_s \tag{5}$$

5. ENVIRONMENTAL DAMAGES AND BENEFITS

The calculation of regional benefits, *i.e.* the avoided environmental damages, is done in four steps. First, the functional form of the damage function is determined, giving global damages in year t as a function of the stock of greenhouse gases. Second, the global benefit function is derived from the damage function. Third, the parameter values for this global benefit function are determined. Fourth, the global benefits are disaggregated over the regions.

5.1. Global environmental damages

As a starting point for the global damage function we look at the specification of damages in DICE (Nordhaus, 1994). In the DICE model, damages are a function of the change in temperature:

$$D_t = \gamma_D \cdot \left[\frac{\Delta T_t}{3}\right]^2 \cdot Y_t \tag{6}$$

Where D_t = total damages in billion US$

γ_D = impact on GDP due to an increase in temperature of 3°C

ΔT_t = Increase in world temperature

Y_t = Global GDP

However, for the STACO model we need damages as a function of the stock of CO_2. Following Germain and Van Steenberghe (2001), we use an approximation of the full climate module and specify temperature change directly as a function of the stock of CO_2:

$$\Delta T_t = \eta \cdot \ln\left(\frac{M_t}{M_{pre-ind}}\right) \tag{7}$$

Where η is a parameter. Equation (7) can be substituted into equation (6) to give global damages as a function of the stock of CO_2:

$$D_t = \left(\frac{\gamma_D}{9}\right) \cdot \left[\eta \cdot \ln\left(\frac{M_t}{M_{pre-ind}}\right)\right]^2 \cdot Y_t \tag{8}$$

The value of η can be calculated by looking at the damages for a doubling of the stock. The DICE model assumes a doubling of the CO_2 concentrations (*i.e.* stock = 2*$M_{pre-ind}$ = 2*590 = 1180 Gton) leads to an increase in temperature of 3 degrees. Then $\eta = \frac{3}{\ln(2)}$, and γ_D gives the damages in percentages of GDP for a doubling of concentrations:

$$D_t = \left[\frac{1}{\ln(2)} \cdot \ln\left(\frac{M_t}{M_{pre-ind}}\right)\right]^2 \cdot (\gamma_D \cdot Y_t) \tag{9}$$

Note that in the damage function described above, the dependence of the stock of CO2 on the abatement level was not made explicit. The undiscounted damages in year *t* are a function of the abatement levels for all periods up to *t*: $D_t\left(M_t\left(q_{2011},\ldots,q_t\right)\right)$.

5.2. Transforming the damage function into a benefit function

The STACO model uses a benefit function instead of a damage function, where benefits equal avoided damages. The model maximizes the payoff from abatement, *i.e.* net benefits. If alternatively damages were used, the optimization procedure would minimize total costs, *i.e.* damages plus abatement costs. It is straightforward to show that both approaches lead to the same model outcomes, as total benefits equal avoided total damages by definition.

Benefits are calculated as avoided damages[5]:

$$B_t(q) = D_t\left(M_t(0)\right) - D_t\left(M_t(q)\right) \tag{10}$$

5.3. Empirical calibration of the global damage and benefit function

The next step in calibrating the benefit function is the choice of the scale parameter γ_D. Nordhaus uses a value of 0.0133 (*i.e.* 1.33% of GDP) for this parameter. It is however well known that the DICE estimate of environmental damages is rather low[6]. Therefore, we scale the global damage

[5] Note that the subscript for time periods for abatement levels *q* is dropped for readability [??].

[6] This is the case for both the original and later versions of the DICE model.

function using the more detailed study of Tol (1997). The estimated year of doubling of concentrations is roughly in line with Tol (doubling of concentrations occurs after just over 60 years). Tol (1997) presents an estimate of total damage costs of 2.7% of GDP for a doubling of CO2-concentrations: $\gamma_D = 0.027$. The associated level of (undiscounted) GDP, Y_{2061}, is calibrated to the DICE model and equals 70284 bln $[7].

The only unknown left in the damage function is the abatement level q; we turn to this later.

5.4. Regional disaggregation of the benefits

The benefit function only gives the benefits for the world. In the STACO model, we use a benefit function per region. We introduce s_i to denote the share of annual damage costs for region i. The distribution of the benefits over the regions is based on Fankhauser (1995) and Tol (1997); their absolute numbers and the associated shares are represented in Tables 1 and 2.

Table 1. Distribution of annual damage costs over different regions in absolute amounts and shares according to Fankhauser (1995).

Region	Fankhauser (1995)[1]
	bln $ *(%)*
USA	61.0 *(22.6%)*
European Union	63.6 *(23.6%)*
Other OECD	55.8 *(20.7%)*
Former Soviet Union	18.2 *(6.8%)*
China	16.7 *(6.2%)*
Rest of the world	54.2 *(20.1%)*
WORLD	269.5 *(100%)*
Subtotal OECD	180.4 *(66.9%)*
Subtotal non-OECD	89.1 *(33.1%)*

[7] This figure was obtained from the DICE model using a weighted average of the estimated GDP level in 2055 and 2065 and then adjusting the 1988 prices to 1985 prices (using US-OMB, 2003) in order to make the numbers comparable with the abatement costs. Alternatively, we could use a whole path of GDP levels to estimate the damage function, but this would complicate the interpretation of parameter γ_D.

Table 2. Distribution of annual damage costs over different regions in absolute amounts and shares according to Tol (1997).

Region	Tol (1997)
	bln $ *(%)*
OECD-America	68.4 *(13.1%)*
OECD-Europe	35.3 *(6.7%)*
OECD-Pacific	62.9 *(12.0%)*
Other Europe	-11.6 *(-2.2%)*
Middle East	15.9 *(3.0%)*
Latin America	109.9 *(21.0%)*
South & Southeast Asia	134.3 *(25.6%)*
Centrally planned Asia	69.6 *(13.3%)*
Africa	39.1 *(7.5%)*
WORLD	523.8 *(100%)*
Subtotal OECD	166.6 *(31.8%)*
Subtotal non-OECD	357.2 *(68.2%)*

Comparing the numbers of Fankhauser and Tol is not straightforward. For instance, Fankhauser's numbers are based on purchasing-power-parity exchange rates, while Tol uses market exchange rates. Moreover, the categorisation in regions varies significantly between Fankhauser and Tol. For instance, Tol provides numbers for USA and Canada together, while Fankhauser puts Canada in the category "Other OECD" (OOE). Therefore, it was unavoidable to make some *ad-hoc* decisions.

Two alternative sets of regional shares were constructed. The first alternative ("STACO calibration I") is primarily based on the estimates of Fankhauser, which have relatively high shares for the OECD regions and relatively low shares for the non-OECD regions. As Fankhauser does not provide information for all regions in the STACO model, some additional assumptions have to be made. The second ("STACO calibration II"), is based as far as possible on Tol's estimates; again additional assumptions are required. The resulting calibrated absolute regional damage costs and the associated shares for both calibration alternatives are given in Table 3 and will be discussed below.

Table 3. Distribution of annual damage costs over different regions in absolute amounts and shares according to the two calibration alternatives.

Region	STACO calibration I	STACO calibration II
	bln \$ *(%) =* s_i	bln \$ *(%) =* s_i
1 USA	61.0 (22.6%)	64.8 (12.4%)
2 JPN	46.5 (17.3%)	59.6 (11.4%)
3 EEC	63.6 (23.6%)	33.5 (6.4%)
4 OOE	9.3 (3.5%)	8.7 (1.7%)
5 EET	3.5 (1.3%)	6.8 (1.3%)
6 FSU	18.2 (6.7%)	18.2 (3.5%)
7 EEX	8.1 (3.0%)	15.9 (3.0%)
8 CHN	16.7 (6.2%)	32.5 (6.2%)
9 IND	13.4 (5.0%)	89.5 (17.1%)
10 DAE	6.7 (2.5%)	44.8 (8.5%)
11 BRA	4.1 (1.5%)	27.5 (5.2%)
12 ROW	18.3 (6.8%)	122.0 (23.3%)
WORLD	269.4 *(100%)*	523.8 *(100%)*
Subtotal OECD	180.4 *(66.9%)*	166.6 *(31.8%)*
Subtotal non-OECD	89.1 *(33.1%)*	357.2 *(68.2%)*

In *STACO calibration I*, the data for USA are directly taken from Fankhauser. Damages for JPN are disaggregated from OOE assuming 5 times as much damages in Japan as in the other countries (roughly based on the share of Japan in total GDP of OOE+JPN). For EEC, the European Union estimate of Fankhauser is used. The damages for OOE are 1/6th of Fankhauser's estimate for Other OECD. The total damages in the OECD for STACO calibration I match Fankhauser: 180.4 bln US\$.

For EET, no damage estimate is available. Based on the share of this region in global GDP, we assume the damage share of this region to be 1.3% of global damages. Fankhauser provides an estimate for FSU that can be directly used. The estimate of the damage share for EEX is based on Tol's estimate for Middle East, 3.0%, as Fankhauser does not provide an estimate. The absolute level of damages for this region is calculated using this share of 3% in global damages. The value for CHN is directly taken from Fankhauser. For the last 4 regions, IND, DAE, BRA and ROW, Fankhauser does not provide sufficient regional information. To match global damages with Fankhauser's estimate, the sum of the damages for these 4 regions have to equal 42.5 bln \$. These damages are attributed to the 4 separate regions using their relative shares calculated in the STACO calibration II as discussed below.

In *STACO calibration II*, the estimates of Tol are the basis for our numbers. Tol provides estimates for Northern America (USA and Canada), a wider range of countries in Europe and Pacific OECD countries. The shares for USA, JPN and EEC are derived by rescaling Tol's estimates such that total OECD damages equal 166.6 bln US\$. The share of STACO region OOE is taken from the first calibration alternative and equals 5.2% of total OECD damages.

The calibration of EET in alternative II is based on the same assumption as in alternative I: 1.3% of global damages. For FSU, the negative estimate of Tol is rejected and the absolute damage estimate of Fankhauser is used. The EEX estimate can be directly taken from Tol. For CHN, the share of the region in global damages is taken from calibration alternative I. Tol's estimate for Asia is divided into two-thirds for IND and one-third for DAE. Tol gives damages for the whole of Latin-America, and we assume that the contribution of Brazil is 25% of that estimate. Finally, the share of ROW is calibrated such that the total damage estimate for non-OECD countries matches with Tol. Note that the share of this region is much higher than in Tol's estimate, since in the STACO classification, ROW also includes all centrally planned Asian countries except China (*i.e.* Vietnam, Laos, Mongolia, North Korea) and all Latin-American countries except Brazil.

Based on these shares, the regional total benefits equal the share of the region times global benefits:

$$TB_{it}\left(q\right) = s_i \cdot B_t\left(q\right) \tag{11}$$

6. ABATEMENT COSTS

The source for the abatement cost function is the EPPA model as reported by Ellerman and Decaux (1998). Let x_{it} be the annual abatement level in Megaton[8] for region i in year t and $AC^E_{it}(x_i)$ denote the associated undiscounted abatement costs according to Ellerman and Decaux (1998). Then,

$$AC^E_{it}\left(x_{it}\right) = \tfrac{1}{3} \cdot a^E_i \cdot \left(x_{it}\right)^3 + \tfrac{1}{2} \cdot b^E_i \cdot \left(x_{it}\right)^2 \tag{12}$$

7. THE PAY-OFF FUNCTION

Pay-off in region i, $\pi_i\left(q_i, q\right)$, as a function of abatement in region i and of the global level of abatement (all in billion US$) is defined as the regional benefits of global abatement minus the regional costs of regional abatement:

$$\pi_{i,t}\left(q_{i,t}, q\right) = TB_{i,t}\left(q\right) - AC_{i,t}\left(q_{i,t}\right) \tag{13}$$

This pay-off function is the main ingredient for the game-theoretic analysis.

8. AGGREGATING OVER TIME

8.1. Assessing the path of abatement levels over time

Some path of abatement levels has to be specified exogenously in order to convert the dynamic equation (5) to be suitable for a static model. We made the *ad hoc* assumption that absolute abatement levels are constant over time. This has the advantage that abatement costs are also constant over time. If we introduce $q_i = \sum_{t=2011}^{2110} q_{it}$ and $q = \sum_{i \in I} q_i$, then we can express the 2110

[8] The abatement levels are presented as x and not q to emphasise the difference in accounting units: q is in Gigatons, x in Megatons.

stock of CO_2 (expression (5)) as a linear function of the (century) abatement level[9]:

$$M_{2110}(q) = 1585 - 0.429 \cdot q \quad (14)$$

8.2. Discounted benefits

The discounted total benefits for a given level of abatement q equals the net present value of the stream of benefits over the model horizon, *i.e.* discount the benefits for each year to 2010 and then sum these discounted benefits:

$$TB(q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot B_t(q) \right\} \quad (15)$$

The value of the discount rate cannot be taken from data, but to some extent reflects a subjective evaluation of the time preference of consumers. The discount rate is assumed to be positive but rather low, $r = 0.02$. A low discount rate ensures some weight on future damage costs. The discount rate will be subject of sensitivity analysis.

In the relevant range of M_t, *i.e.* between M_{2010} and $M_{2110}(q=0)$, the non-linear function is virtually linear and reduces to

$$\widetilde{TB}_i(q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot s_i \cdot \gamma_D \cdot \left(\frac{1}{\ln(2)} \right)^2 \cdot Y_t \cdot c_B \cdot \left(\frac{M_t(0) - M_t(q)}{M_{pre-ind}} \right) \right\} \quad (16)$$

where c_B denotes the constant slope of the linear approximation (c_B =0.72).

The linear approximation of the non-linear function is shown in Figure 3.

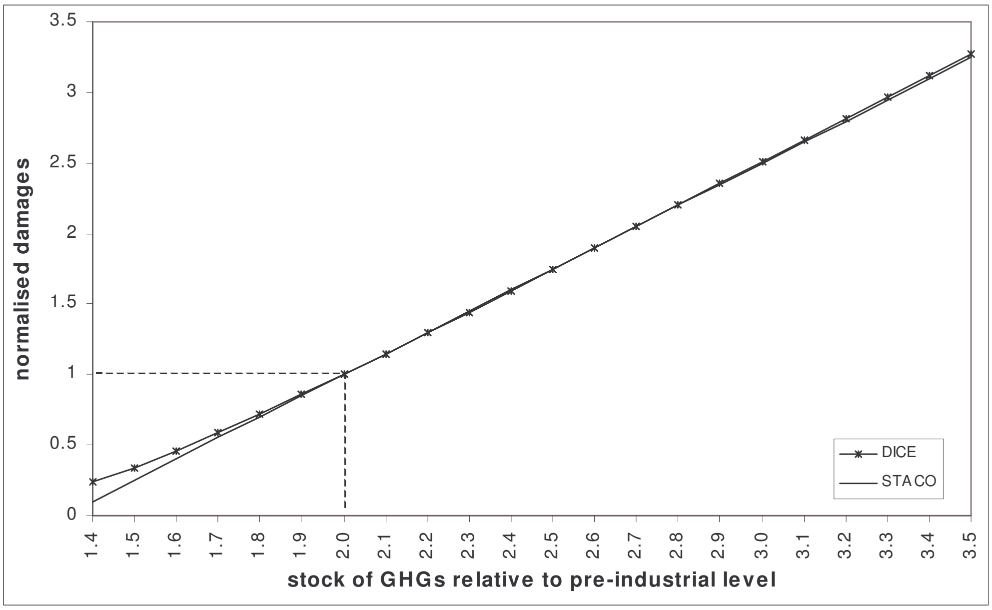

Figure 3. The normalised non-linear damage function based on the DICE model and the linear STACO approximation.

[9] Note that q_i is the *sum* over the periods for region i, not the level for an individual year. The abatement level for individual years equal the century abatement level divided by the number of years: $q_{it} = q_i / 100$.

For sake of simplicity, the current version of the model is calibrated to a single observation for GDP, Y_{2061}, as this is the date of doubling of emissions. Alternatively, one could calibrate the model to a whole path of GDP levels for the entire century; the difference between both alternatives is less than 10 percent.

Equation (16) can be re-arranged to

$$\widetilde{TB}_i(q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot s_i \cdot \gamma_D \cdot C \cdot \left(M_t(0) - M_t(q) \right) \right\} \tag{17}$$

with $C = \left(\frac{1}{\ln(2)} \right)^2 \cdot Y_{2061} \cdot \left(\frac{1}{M_{pre-ind}} \right) \cdot c_B = 178.331$; using (5) gives

$$\widetilde{TB}_i(q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot s_i \cdot \gamma_D \cdot C \cdot \left(\sum_{s=2011}^{t} (1-\delta_M)^{t-s} \cdot 0.64 \cdot q/100 \right) \right\} \tag{18}$$

$\Leftrightarrow$

$$\widetilde{TB}_i(q) = s_i \cdot \gamma_D \cdot C \cdot q \cdot \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot \left(\sum_{s=2011}^{t} (1-\delta_M)^{t-s} \cdot 0.64/100 \right) \right\} \tag{19}$$

$\Leftrightarrow$

$$\widetilde{TB}_i(q) = s_i \cdot \gamma_D \cdot C \cdot q \cdot 7.767 = s_i \cdot 37.40 \cdot q \tag{20}$$

As the stock of greenhouse gases is linear in the abatement level and damages are linear in the stock, it follows that the total benefit function is also linear in abatement level q. Hence, the global marginal benefits from reducing 1 ton of CO2 spread over the 100 years, *i.e.* 10 kg per year for each year between 2010 and 2110, do not depend on the abatement level and equal \$37.40. This figure is in line with results from Plambeck and Hope (1996) who report that what they call their "best" estimate for marginal benefits (in a regional scenario) falls within a range of US\$10-48 per ton of carbon considering a 90% confidence interval[10].

The slope of the benefit function as derived from the DICE model gives smaller marginal benefits: 18.42 \$/ton (this can be calculated by changing the scale parameter γ_D from 0.027 to 0.0133).

8.3. Discounted abatement costs

In addition to the assumption that the level of abatement is constant over time, annual undiscounted abatement costs are also assumed to be constant over time, *i.e.* no efficiency improvements in abatement are accounted for.

We introduce α_i and β_i as parameters for calculating the undiscounted annual abatement cost function in the STACO model as a function of the total abatement over the century (in billion US\$):

[10] Note that Plambeck and Hope's estimate is discounted to 1990, while our estimate is discounted to 2010. Discounting our estimate to 1990 leads to an estimate of marginal benefits of just over 25 \$/ton.

$$AC_{it}\left(q_i\right)=\tfrac{1}{3}\cdot\alpha_i\cdot\left(q_i\right)^3+\tfrac{1}{2}\cdot\beta_i\cdot\left(q_i\right)^2 \tag{21}$$

As absolute abatement levels and undiscounted abatement costs are assumed to be constant over time, the way to reduce 1 Gton in a century is to reduce 1/100 Gton = 10 Mton per year. The main relation that has to hold in our study is that the annual undiscounted costs of reducing 1 Gton in a century are equal to the costs of reducing 10 Mton per year. In other words

$$AC_{it}\left(1\right)=\frac{AC_{it}^{E}\left(10\right)}{1000} \tag{22}$$

The denominator converts the figures used by Ellerman and Decaux to STACO figures: AC_{it} is given in billion \$ whereas AC^{E}_{it} is given in million \$.

Substituting (12) and (21) in (22) gives the following relationship between our parameters and the ones used by Ellerman and Decaux:

$$\tfrac{1}{3}\cdot\alpha_i\cdot\left(1\right)^3+\tfrac{1}{2}\cdot\beta_i\cdot\left(1\right)^2=\left\{\tfrac{1}{3}\cdot a_i^E\cdot\left(10\right)^3+\tfrac{1}{2}\cdot b_i^E\cdot\left(10\right)^2\right\}\Big/1000 \tag{23}$$

and hence

$$\alpha_i=a_i^E \text{ and } \beta_i=0.1\cdot b_i^E \tag{24}$$

In Ellerman and Decaux (1998), region OOE, *i.e.* other OECD countries, which includes mainly Canada and Australia, has a negative value for α_{OOE}. This leads to technical problems in the model and hence the parameters for the region OOE are recalibrated under the assumption that $\alpha_{\text{OOE}} = 0$. The value for β_{OOE} is chosen such that a best fit is achieved for the discounted marginal abatement cost curve as used in the STACO model.

The total discounted abatement costs over the full model horizon can be calculated as the net present value of the stream of abatement costs over time

$$TAC_i\left(q_i\right)=\sum_{t=2011}^{2110}\left\{\left(1+r\right)^{-(t-2010)}\cdot AC_{it}\left(q_i\right)\right\} \tag{25}$$

Where r denotes the discount rate. We use r=0.02 as in the discounting of benefits. Since AC_{it} does not depend on time, equation (25) boils down to

$$TAC_i\left(q_i\right)=43.1\cdot\left\{\tfrac{1}{3}\cdot\alpha_i\cdot\left(q_i\right)^3+\tfrac{1}{2}\cdot\beta_i\cdot\left(q_i\right)^2\right\} \tag{26}$$

Finally, the discounted marginal abatement costs

$$MAC_i\left(q_i\right)=43.1\cdot\left\{\alpha_i\cdot\left(q_i\right)^2+\beta_i\cdot q_i\right\} \tag{27}$$

The resulting MAC curves are represented in *Figure 4*. The curves are truncated at the point where abatement levels exceed 100 times the emissions in 2010, as this is the upper bound on abatement (see above). The x-axis gives the total emission reduction in Gigaton for the entire century: q_i in equation (27) above. The y-axis gives the associated discounted marginal abatement costs as calculated using equation (27).

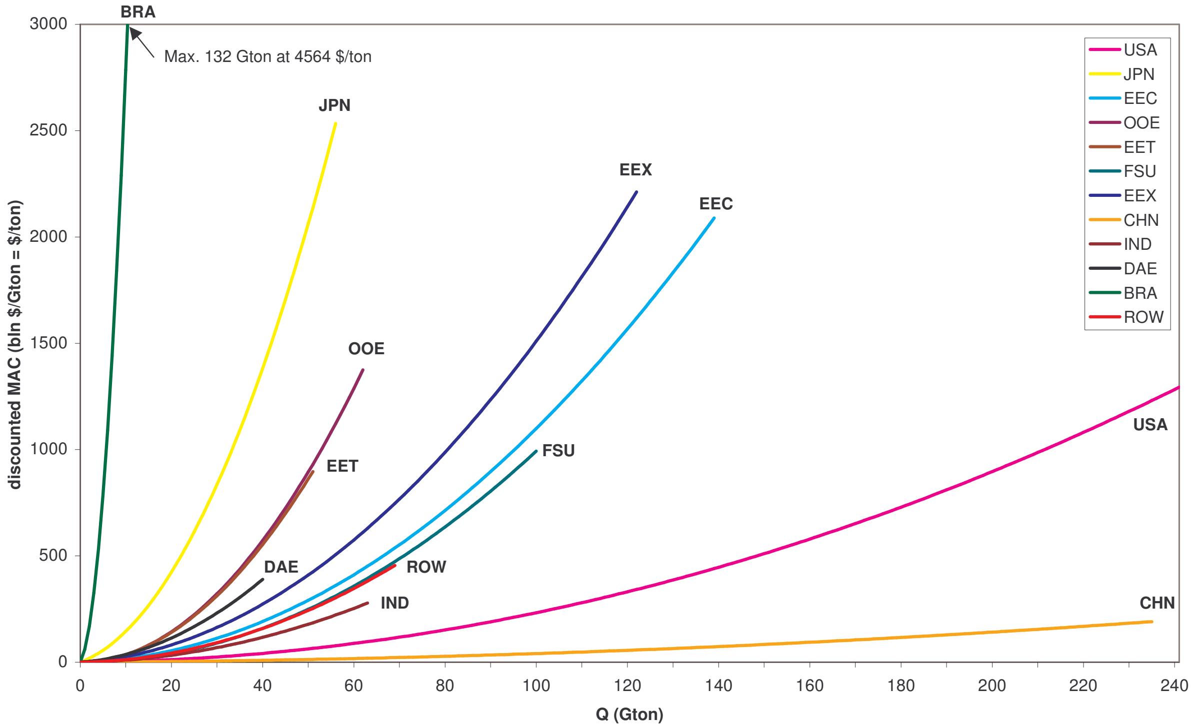

Figure 4. Discounted marginal abatement cost functions for regions as calibrated in the STACO model (own calculations based on data from Ellerman and Decaux, 1998).

8.4. Discounted pay-offs

The STACO model captures the net present value of the stream of pay-offs generated between 2011 and 2110. The pay-off function for the discounted costs and benefits is as follows:

$$\pi_i(q_i, q) = TB_i(q) - TAC_i(q_i) \qquad (28)$$

Note that this function can also be derived by discounting the stream of pay-offs over time:

$$\pi_i(q_i, q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot \pi_{it}(q_{i,t}, q) \right\} \qquad (29)$$

(28) and (29) give identical results.

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APPENDIX I. PARAMETERS AND VARIABLES

Parameter values

Symbol	Description	Value	Units	Source
E_{2010}	Global CO_2 emissions in year 2010	11.96	Gton	DICE model
$E_{i,2010}$	Regional CO_2 emissions in year 2010	see table below	Gton	own calculation based on the EPPA model
d_E	Annual growth in global CO_2 emissions	0.153	Gton	own calculation
$M_{pre\text{-}ind}$	pre-industrial level of CO_2-stock	590	Gton	DICE model
M_{2010}	Stock of CO_2 in 2010, starting point for calculations	835	Gton	DICE model
r	Annual discount rate	0.02	n/a	assumption
s_i	Share of region *i* in global benefits	see table below	n/a	own calculation
α_i	Parameter for abatement cost function	see table below	n/a	EPPA model
β_i	Parameter for abatement cost function	see table below	n/a	EPPA model
γ_D	Scale parameter for damages and benefits (share of GDP)	0.027	n/a	Tol

Region (*i*)	$E_{i,2010}$ (share of total)	s_i (calibration I)	s_i (calibr. II)	α_i	β_i
1 USA	0.202	0.226	0.124	0.0005	0.00398
2 JPN	0.047	0.173	0.114	0.0155	0.18160
3 EEC	0.117	0.236	0.064	0.0024	0.01503
4 OOE	0.052	0.035	0.017	0.0083	0
5 EET	0.043	0.013	0.013	0.0079	0.00486
6 FSU	0.084	0.067	0.035	0.0023	0.00042
7 EEX	0.102	0.030	0.030	0.0032	0.03029
8 CHN	0.197	0.062	0.062	0.00007	0.00239
9 IND	0.053	0.050	0.171	0.0015	0.00787
10 DAE	0.034	0.025	0.085	0.0047	0.03774
11 BRA	0.011	0.015	0.052	0.5612	0.84974
12 ROW	0.058	0.068	0.233	0.0021	0.00805
WORLD	Σ=1	$\sum s_i = 1$	$\sum s_i = 1$		

Variables

Symbol	Description	Unit
E_{it}	Emissions in region *i* in year *t*	Gton
$M_t(q_t)$	Stock of CO2 in period *t*	Gton
q_{it}	Abatement in year *t* in region i	Gton
q_t	Abatement in year *t*, world total ($q_t = \sum_i q_{it}$)	Gton
q_i	Abatement over full period 2010-2110 in region *i* ($q_i = \sum_{t=2011}^{2110} q_{it}$)	Gton
q	Abatement over full period 2010-2110, world total ($q = \sum_{i \in I} q_i$)	Gton
$D_t(M_t)$	Undiscounted global damages in year *t* for stock M_t	bln $
$B_t(q)$	Undiscounted global benefits in year *t* for abatement level *q*	bln $
$TB(q)$	Discounted global benefits over full period 2010-2110 for abatement level *q*	bln $
$TB_i(q)$	Discounted benefits for region *i* over full period 2010-2110 for abatement level *q*	bln $
MB_i	Discounted marginal benefits (for all abatement levels)	$ / ton
	N.B. 1 ton over 100 years implies 10 kg per year	
$AC_{it}(q_i)$	Undiscounted abatement costs for region *i* for period *t* for abatement level q_i	bln $
$TAC_i(q_i)$	Discounted abatement costs for region *i* over full period 2010-2110 for abatement level q_i	bln $
$MAC_i(q_i)$	Discounted marginal benefits for abatement level q_i	$ / ton
$\pi_i(q_i, q)$	Net pay-off from abatement for region *i* over full period 2010-2110 for regional abatement level q_i and global abatement level *q*	bln $

APPENDIX II. EQUATIONS IN THE STACO MODEL

Annual equations

Annual global stock of CO_2 (Mt) as function of annual global abatement efforts (q)

$$M_t\left(q_{2011},\ldots,q_t\right) = M_{pre-ind} + \left(1-\delta_M\right)^{(t-2010)} \cdot \left(M_{2010} - M_{pre-ind}\right) + \sum_{s=2011}^{t}\left(\left(1-\delta_M\right)^{t-s} \cdot 0.64 \cdot \sum_{i\in I}\left(E_{i,s} - q_{i,s}\right)\right) \tag{30}$$

(based on the DICE model)

Annual regional damages as function of global stock of CO_2

$$D_{i,t}\left(M_t\right) = s_i \cdot \left(\frac{\gamma_D}{9}\right) \cdot \left[\eta \cdot \ln\left(\frac{M_t}{M_{pre-ind}}\right)\right]^2 \cdot Y_t \tag{31}$$

(based on the DICE model and Germain and Van Steenberghe, 2001)

Annual regional benefits as function of annual global abatement efforts

$$B_{i,t}\left(q\right) = D_{i,t}\left(M_t\left(0\right)\right) - D_{i,t}\left(M_t\left(q\right)\right) \tag{32}$$

(definition)

Annual regional abatement costs as function of annual regional abatement efforts

$$AC_{i,t}\left(q_{i,t}\right) = \tfrac{1}{3} \cdot \alpha_i^a \cdot \left(q_{i,t}\right)^3 + \tfrac{1}{2} \cdot \beta_i^a \cdot \left(q_{i,t}\right)^2 \tag{33}$$

(based on the EPPA model)

Annual regional pay-off function as function of annual abatement efforts

$$\pi_{i,t}\left(q_{i,t}, q\right) = B_{i,t}\left(q\right) - AC_{i,t}\left(q_{i,t}\right) \tag{34}$$

(definition)

Aggregation over time

Assume stationary abatement efforts

$$q_{i,t} = q_i / 100 \tag{35}$$

(assumption)

Discounted regional damages as function of global century abatement efforts[11]

$$TD_i(q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot D_{it}(q) \right\} \tag{36}$$

$\Leftrightarrow$

$$TD_i(q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot s_i \cdot \gamma_D \cdot Y_t \cdot \left[\frac{1}{\ln(2)} \cdot \ln\left(\frac{M_t(q)}{M_{pre-ind}} \right) \right]^2 \right\} \tag{37}$$

Discounted regional benefits as function of global century abatement efforts

$$TB_i(q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot B_{i,t}(q) \right\} \tag{38}$$

$\Leftrightarrow$

$$\widetilde{TB}_i(q) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot s_i \cdot \gamma_D \cdot \left(\frac{1}{\ln(2)} \right)^2 \cdot Y_t \cdot c_B \cdot \left(\frac{\sum_{s=2011}^{t} (1-\delta_M)^{t-s} \cdot 0.64 \cdot q/100}{M_{pre-ind}} \right) \right\} \tag{39}$$

Discounted regional abatement costs as function of regional century abatement efforts

$$TAC_i(q_i) = \sum_{t=2011}^{2110} \left\{ (1+r)^{-(t-2010)} \cdot AC_{it}(q_i) \right\} \tag{40}$$

$\Leftrightarrow$

$$TAC_i(q_i) = \tfrac{1}{3} \cdot \alpha_i \cdot (q_i)^3 + \tfrac{1}{2} \cdot \beta_i \cdot (q_i)^2 \tag{41}$$

Discounted regional pay-off function as function of century abatement efforts

$$\pi_i(q_i, q) = TB_i(q) - TAC_i(q_i) \tag{42}$$

Marginal benefits and costs

Discounted regional marginal damages

$$MD_i(q) = \frac{\partial TD_i(q)}{\partial q} \tag{43}$$

[11] Note that the parameter values in these functions have to be recalibrated to take account of the century versus annual abatement efforts; this holds for the damage, benefit and abatement cost functions.

Discounted regional marginal benefits

$$MB_i(q) = \frac{\partial TB_i(q)}{\partial q} \tag{44}$$

Discounted linearised regional marginal benefits[12]

$$\widetilde{MB}_i(q) = \frac{\partial \widetilde{TB}_i(q)}{\partial q} = s_i \cdot \left(\frac{\gamma_D}{9}\right) \cdot \eta^2 \cdot \gamma_B \cdot \sum_{t=2011}^{2110} \left\{ \left(\frac{\sum_{s=2011}^{t} (1-\delta_M)^{t-s} \cdot 0.64/100}{M_{pre-ind}} \right) \cdot Y_t \right\} \tag{45}$$

Discounted regional marginal abatement costs

$$MAC_i(q_i) = \frac{\partial TAC_i(q_i)}{\partial q_i} \tag{46}$$

[12] Note that this function does not depend on abatement efforts, but is constant.